Practice Problems

1. Sketch $f(x) = 2x^3 - 3x^2 + 12x + 9$.

Find the derivatives of:

2. $y = 7$
3. $y = 2x$
4. $y = 2(x+1)$
5. $y = x^3$
6. $y = \sin(2x)$
7. $y = \cos(4x^3)$
8. $y = \tan(x)$
9. $y = \sec(2+x)$
10. $y = \cot(x)$
11. $y = \csc(2x^2 - 3x + 2)$
12. $y = \arcsin(x)$
13. $y = \arctan(x^2)$
14. $y = \operatorname{arcsec}(x^4)$
15. $y = \arccos(x)$
16. $y = \operatorname{arccot}(x+2)$
17. $y = \operatorname{arccsc}(10x)$
18. $y = \ln(x)$
19. $y = e^{2x-3}$
20. $y = a^{4x}$
21. $y = \log_a 2x$
22. $y = x^2 + 4x^3$
23. $y = (x^2 + 2x)(x+1)$
24. $y = \dfrac{x^3 - 3x^2 + 4}{x^2}$
25. $y = (3x - 2x^2)^3$
26. $y^3 + y^2 - 5y - x^2 = -4$.
27. $x^3 + y^3 = 4xy + 1$

Answer Key.

① Using the 8 steps outlined in section 3.6:

ⓐ find domain and range

Domain: $x = \mathbb{R}$

Range: $y = \mathbb{R}$.

ⓑ find intercepts.

$(0, 9) \Rightarrow$ y intercept

$(-0.166, 0) \Rightarrow$ x-intercept

ⓒ find symmetry

No symmetry.

ⓓ find asymptotes

No asymptotes.

ⓔ Find the critical points

$f'(x) = 6x^2 - 6x + 12 = 0.$

$6(x^2 - x + 2) = 0.$

No critical points, meaning no slope direction change.

ⓕ Find Potential Inflection Points.

$f''(x) = 12x - 6 = 0.$

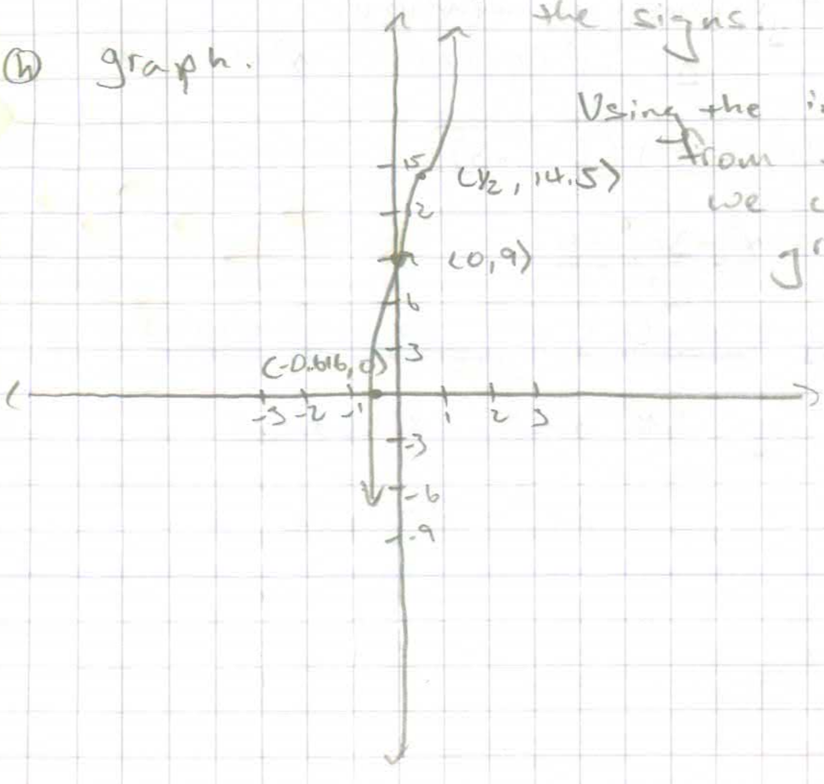
$x = 0.5.$

ⓖ Sign Chart.

	$x < \frac{1}{2}$	$\frac{1}{2}$	$x > \frac{1}{2}$
$f(x)$		14.5	
$f'(x)$	+	+	+
$f''(x)$	−	0	+
	Increasing Concave Down	IP	Increasing Concave up.

$\Rightarrow$ we plug in numbers greater than or lesser than to find the signs.

ⓗ graph.



Using the information from the sign chart, we can sketch the graph.

② $y = 7$
$y' = 0$

③ $y = 2x$
$y' = 2$

④ $y = 2(x+1)$
$y' = 2$

⑤ $y = x^3$
$y' = 3x^2$

⑥ $y = \sin(2x)$
$y' = \cos(2x) \cdot 2$
$y' = 2\cos(2x)$

⑦ $y = \cos(4x^3)$
$y' = -\sin(4x^3) \cdot 12x^2$
$y' = -12x^2 \cdot \sin(4x^3)$

⑧ $y = \tan(x)$
$y' = \sec^2(x)$

⑨ $y = \sec(2+x)$
$y' = \sec(2+x)\tan(2+x)$

⑩ $y = \cot(x)$
$y' = -\csc^2(x)$

⑪ $y = \csc(2x^3 - 3x + 2)$
$y' = -\csc(2x^3-3x+2)\tan(2x^3-3x+2)\boxed{(6x^2-3)}$

⑫ $y = \arcsin(x)$
$y' = \frac{1}{\sqrt{1-x^2}}$

⑬ $y = \arctan(x^2)$
$y' = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$

⑭ $y = \operatorname{arcsec}(x^4)$
$y' = \frac{1}{|x^4|\sqrt{(x^4)^2-1}} \cdot 4x^3 = \frac{4x^3}{|x^4|\sqrt{x^8-1}}$

⑮ $y = \arccos(x)$
$y' = -\frac{1}{\sqrt{1-x^2}}$

⑯ $y = \operatorname{arccot}(x+2)$
$y' = -\frac{1}{\sqrt{1+(x+2)^2}}$

⑰ $y = \operatorname{arcsec}(10x)$
$y' = -\frac{1}{|10x|\sqrt{(10x)^2-1}} \cdot 10 = \frac{10}{|10x|\sqrt{100x^2-1}}$

⑱ $y = \ln(x)$
$y' = \frac{1}{x}$

⑲ $y = e^{2x-3}$
$y' = e^{2x-1} \cdot 2$
$y' = 2e^{2x-1}$

⑳ $y = a^{4x}$
$y' = (\ln 4 \cdot a^{4x})4$

㉑ $y = \log_a 2x$

$y' = \frac{1}{\ln a \cdot 2x} \cdot 2 = \frac{2}{\ln(a) \cdot 2x}$

㉒ $y = x^2 + 4x^3$
$y' = 2x + 12x^2$

㉓ $y = (x^2+2x)(x+1)$
$y' = (2x+2)(x+1) + (x^2+2x)(1)$
$y' = 2x^2 + 2x + 2x + 1 + x^2 + 2x$
$y' = 3x^2 + 6x + 1$

㉔ $y = \frac{x^3 - 3x^2 + 4}{x^2}$
$y' = \frac{(3x^2-6x)(x^2) - (x^3-3x^2+4)(2)}{(x^2)^2}$
$y' = \frac{3x^4 - 6x^3 - 2x^3 + 6x^2 - 8}{x^4}$
$y' = \frac{3x^4 - 8x^3 + 6x^2 - 8}{x^4}$

㉕ $y = (3x-2x^2)^3$
$y' = 3(3x-2x^2)^2 \cdot (3-4x)$

(26) $y^3 + y^2 - 5y - x^2 = -4$.

$\frac{dy}{dx}$